

**Time limit:** 15 minutes.

**Instructions:** This tiebreaker contains 3 short answer questions. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

**No calculators.**

1. Regular hexagon  $NOSAME$  with side length 1 and square  $UDON$  are drawn in the plane such that  $UDON$  lies outside of  $NOSAME$ . Compute  $[SAND] + [SEND]$ , the sum of the areas of quadrilaterals  $SAND$  and  $SEND$ .
2. Let  $\triangle A_0B_0C_0$  be an equilateral triangle with area 1, and let  $A_1, B_1, C_1$  be the midpoints of  $\overline{A_0B_0}$ ,  $\overline{B_0C_0}$ , and  $\overline{C_0A_0}$ , respectively. Furthermore, set  $A_2, B_2, C_2$  as the midpoints of segments  $\overline{A_0A_1}$ ,  $\overline{B_0B_1}$ , and  $\overline{C_0C_1}$  respectively. For  $n \geq 1$ ,  $A_{2n+1}$  is recursively defined as the midpoint of  $\overline{A_{2n}A_{2n-1}}$ , and  $A_{2n+2}$  is recursively defined as the midpoint of  $\overline{A_{2n+1}A_{2n-1}}$ . Recursively define  $B_n$  and  $C_n$  the same way. Compute the value of  $\lim_{n \rightarrow \infty} [A_nB_nC_n]$ , where  $[A_nB_nC_n]$  denotes the area of triangle  $\triangle A_nB_nC_n$ .
3. Right triangle  $\triangle ABC$  with its right angle at  $B$  has angle bisector  $\overline{AD}$  with  $D$  on  $\overline{BC}$ , as well as altitude  $\overline{BE}$  with  $E$  on  $\overline{AC}$ . If  $\overline{DE} \perp \overline{BC}$  and  $AB = 1$ , compute  $AC$ .