

Time limit: 15 minutes.

Instructions: This tiebreaker contains 3 short answer questions. All answers are positive integers. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

No calculators.

1. Compute the smallest positive integer n such that $\frac{n}{2}$ is a perfect square and $\frac{n}{3}$ is a perfect cube.
 2. On a certain planet, the alien inhabitants are born without any arms, legs, or noses. Every year, on their birthday, each alien randomly grows either an arm, a leg, or a nose, with equal probability for each. After its sixth birthday, the probability that an alien will have at least 2 arms, at least 2 legs, and at least 1 nose on the day is $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.
 3. Three distinct integers a_1, a_2, a_3 between 1 and 21, inclusive, are selected uniformly at random. The probability that the greatest common factor of $a_i - a_j$ and 21 is 7 for some positive integers i and j , where $1 \leq i \neq j \leq 3$, can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.
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