

**Time limit:** 15 minutes.

**Instructions:** This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

**No calculators.**

1. We inscribe a circle  $\omega$  in equilateral triangle  $ABC$  with radius 1. What is the area of the region inside the triangle but outside the circle?
2. Define the inverse of triangle  $ABC$  with respect to a point  $O$  in the following way: construct the circumcircle of  $ABC$  and construct lines  $AO, BO,$  and  $CO$ . Let  $A'$  be the other intersection of  $AO$  and the circumcircle (if  $AO$  is tangent, then let  $A' = A$ ). Similarly define  $B'$  and  $C'$ . Then  $A'B'C'$  is the inverse of  $ABC$  with respect to  $O$ . Compute the area of the inverse of the triangle given in the plane by  $A(-6, -21), B(-23, 10), C(16, 23)$  with respect to  $O(1, 3)$ .
3. We say that a quadrilateral  $Q$  is *tangential* if a circle can be inscribed into it, i.e. there exists a circle  $C$  that does not meet the vertices of  $Q$ , such that it meets each edge at exactly one point. Let  $N$  be the number of ways to choose four distinct integers out of  $\{1, \dots, 24\}$  so that they form the side lengths of a tangential quadrilateral. Find the largest prime factor of  $N$ .