1. A number is between 500 and 1000 and has a remainder of 6 when divided by 25 and a remainder of 7 when divided by 9. Find the only odd number to satisfy these requirements.

2. If I roll three fair 4-sided dice, what is the probability that the sum of the resulting numbers is relatively prime to the product of the resulting numbers?

3. Suppose we have 2013 piles of coins, with the i^{th} pile containing exactly i coins. We wish to remove the coins in a series of steps. In each step, we are allowed to take away coins from as many piles as we wish, but we have to take the same number of coins from each pile. We cannot take away more coins than a pile actually has. What is the minimum number of steps we have to take?

4. Given $f_1 = 2x - 2$ and $k \geq 2$, define $f_k(x) = f_1(f_{k-1}(x))$ to be a real-valued function of x. Find the remainder when $f_{2013}(2012)$ is divided by the prime 2011.

5. Consider the roots of the polynomial $x^{2013} - 2^{2013} = 0$. Some of these roots also satisfy $x^k - 2^k = 0$, for some integer $k < 2013$. What is the product of this subset of roots?

6. A coin is flipped until there is a head followed by two tails. What is the probability that this will take exactly 12 flips?

7. Denote by $S(a, b)$ the set of integers k that can be represented as $k = a \cdot m + b \cdot n$, for some non-negative integers m and n. So, for example, $S(2, 4) = \{0,\ 2,\ 4,\ 6,\ \dots\}$. Then, find the sum of all possible positive integer values of x such that $S(18, 32)$ is a subset of $S(3, x)$.

8. Let $f(n)$ take in a nonnegative integer n and return an integer between 0 and $n-1$ at random (with the exception being $f(0) = 0$ always). What is the expected value of $f(f(22))$?

9. 2013 people sit in a circle, playing a ball game. When one player has a ball, he may only pass it to another player 3, 11, or 61 seats away (in either direction). If $f(A, B)$ represents the minimal number of passes it takes to get the ball from Person A to Person B, what is the maximal possible value of f?

10. Let σ_n be a permutation of $\{1, \dots, n\}$; that is, $\sigma_n(i)$ is a bijective function from $\{1, \dots, n\}$ to itself. Define $f(\sigma)$ to be the number of times we need to apply σ to the identity in order to get the identity back. For example, f of the identity is just 1, and all other permutations have $f(\sigma) > 1$. What is the smallest n such that there exists a σ_n with $f(\sigma_n) = k$?

P1. Ahuiliztli is playing around with some coins (pennies, nickels, dimes, and quarters). She keeps grabbing k coins and calculating the value of her handful. After a while, she begins to notice that if k is even, she more often gets even sums, and if k is odd, she more often gets odd sums. Help her prove this true! Given k coins chosen uniformly and at random, prove that the probability that the parity of k is the same as the parity of the k coins' value is greater than the probability that the parities are different.

P2. Let p be an odd prime, and let $(p^p)! = mp^k$ for some positive integers m and k. Find in terms of p the number of ordered pairs (m, k) satisfying $m + k \equiv 0 \bmod p$.