Time limit: 60 minutes.
Instructions: This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

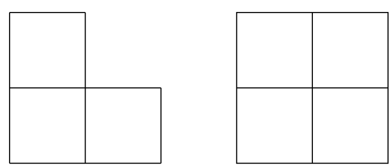
1. If x is 20% of 23 and y is 23% of 20, compute $\frac{x}{y}$.

2. Pablo wants to eat a banana, a mango, and a tangerine, one at a time. How many ways can he choose the order to eat the three fruits?

3. Let a, b, and c be 3 positive integers. If $a + \frac{b}{c} = \frac{11}{6}$, what is the minimum value of $a + b + c$?

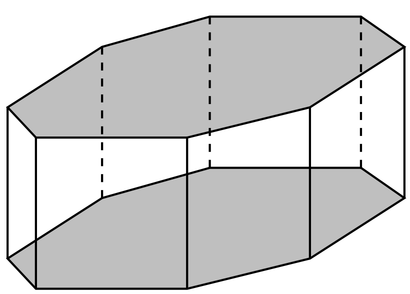
4. A rectangle has an area of 12. If all of its sidelengths are increased by 2, its area becomes 32. What is the perimeter of the original rectangle?

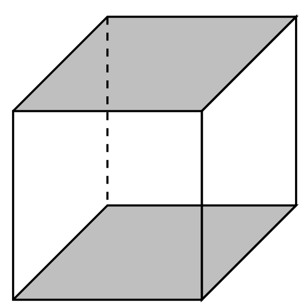
5. Rohit is trying to build a 3-dimensional model by using several cubes of the same size. The model's front view and top view are shown below. Suppose that every cube on the upper layer is directly above a cube on the lower layer and the rotations are considered distinct. Compute the total number of different ways to form this model.



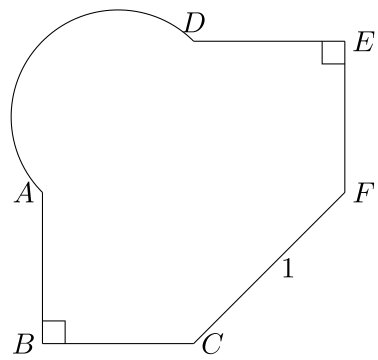
Front View Top View

6. Priscilla has three octagonal prisms and two cubes, none of which are touching each other. If she chooses a face from these five objects in an independent and uniformly random manner, what is the probability the chosen face belongs to a cube? (One octagonal prism and cube are shown below.)

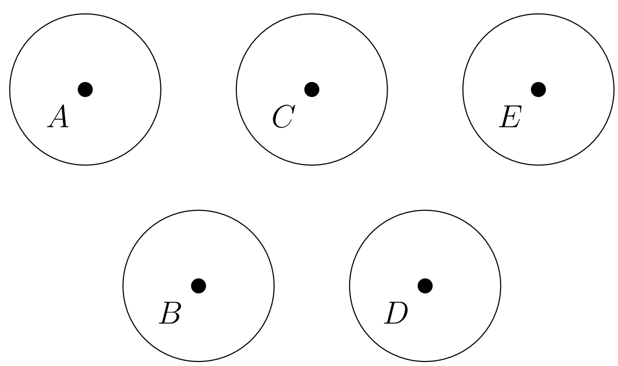




7. Let triangle $\triangle ABC$ and triangle $\triangle DEF$ be two congruent isosceles right triangles where line segments $\overline{AC}$ and $\overline{DF}$ are their respective hypotenuses. Connecting a line segment $\overline{CF}$ gives us a square $ACFD$ but with missing line segments $\overline{AC}$, $\overline{AD}$, and $\overline{DF}$. Instead, A and D are connected by an arc defined by the semicircle with endpoints A and D. If $CF = 1$, what is the perimeter of the whole shape $ABCFED$ (shown on the next page)?



8. There are two moles that live underground, and there are five circular holes that the moles can hop out of. The five holes are positioned as shown in the diagram below, where A, B, C, D, and E are the centers of the circles, $AE = 30$ cm, and congruent triangles $\triangle ABC$, $\triangle CBD$, and $\triangle CDE$ are equilateral. The two moles randomly choose exactly two of the five holes, hop out of the two chosen holes, and hop back in. What is the probability that the holes that the two moles hop out of have centers that are exactly 15 cm apart?



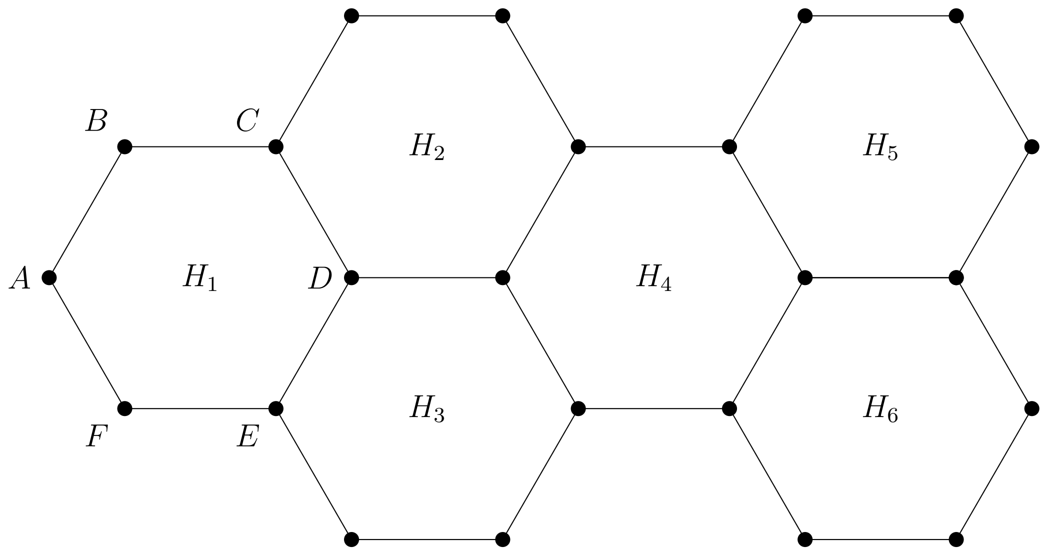
9. Carson is planning a trip for n people. Let x be the number of cars that will be used and y be the number of people per car. What is the smallest value of n such that there are exactly 3 possibilities for x and y so that y is an integer, $x < y$, and exactly one person is left without a car?

10. Iris is eating an ice cream cone, which consists of a hemisphere of ice cream with radius $r > 0$ on top of a cone with height 12 and also radius r. Iris is a slow eater, so after eating one-third of the ice cream, she notices that the rest of the ice cream has melted and completely filled the cone. Assuming the ice cream did not change volume after it melted, what is the value of r?

11. As Natasha begins eating brunch between 11:30 AM and 12 PM, she notes that the smaller angle between the minute and hour hand of the clock is 27 degrees. What is the number of degrees in the smaller angle between the minute and hour hand when Natasha finishes eating brunch 20 minutes later?

12. On a regular hexagon $ABCDEF$, Luke the frog starts at point A; there is food on points C and E and there are crocodiles on points B and D. When Luke is on a point, he hops to any of the five other vertices with equal probability. What is the probability that Luke will visit both of the points with food before visiting any of the crocodiles?

13. 2023 regular unit hexagons are arranged in a tessellating lattice, as follows. The first hexagon $ABCDEF$ (with vertices in clockwise order) has leftmost vertex A at the origin, and hexagons H_2 and H_3 share edges $\overline{CD}$ and $\overline{DE}$ with hexagon H_1, respectively. Hexagon H_4 shares edges with both hexagons H_2 and H_3, and hexagons H_5 and H_6 are constructed similarly to hexagons H_2 and H_3. Hexagons H_7 to H_{2022} are constructed following the pattern of hexagons H_4, H_5, H_6. Finally, hexagon H_{2023} is constructed, sharing an edge with both hexagons H_{2021} and H_{2022}. Compute the perimeter of the resulting figure.



14. Aditya's favorite number is a positive two-digit integer. Aditya sums the integers from 5 to his favorite number, inclusive. Then, he sums the next 12 consecutive integers starting after his favorite number. If the two sums are consecutive integers and the second sum is greater than the first sum, what is Aditya's favorite number?

15. The 100^{th} anniversary of BMT will fall in the year 2112, which is a palindromic year. Compute the sum of all years from 0000 to 9999, inclusive, that are palindromic when written out as four-digit numbers (including leading zeros). Examples include 2002, 1991, and 0110.

16. Points A, B, C, D, and E lie on line r, in that order, such that $DE = 2DC$ and $AB = 2BC$. Let M be the midpoint of segment $\overline{AC}$. Finally, let point P lie on r such that $PE = x$. If $AB = 8x$, $ME = 9x$, and $AP = 112$, compute the sum of the two possible values of CD.

17. A parabola $y = x^2$ in the xy-plane is rotated $180°$ about a point (a, b). The resulting parabola has roots at $x = 40$ and $x = 48$. Compute $a + b$.

18. Susan has a standard die with values 1 to 6. She plays a game where every time she rolls the die, she permanently increases the value on the top face by 1. What is the probability that, after she rolls her die 3 times, there is a face on it with a value of at least 7?

19. Let N be a 6-digit number satisfying the property that the average value of the digits of N^4 is 5. Compute the sum of the digits of N^4.

20. Let O_1, O_2,..., O_8 be circles of radius 1 such that O_1 is externally tangent to O_8 and O_2 but no other circles, O_2 is externally tangent to O_1 and O_3 but no other circles, and so on. Let C be a circle that is externally tangent to each of $O_1, O_2, O_3, \ldots, O_8$. Compute the radius of C.