1. The average of all ten students was 28. So the sum of their scores was $28 \times 10 = 280$. The average of five students was 15, so the sum of their scores was $15 \times 5 = 75$.. So the sum of the scores of the remaining 5 students is $280 - 75 = 205$. Hence the average of their scores is $\frac{205}{5} = \boxed{41}$.

2. Note that $a \otimes b = (a+b)^2$ The calculations:

$$(-5 \otimes 7) \otimes 4 = (7-5)^2 \otimes 4$$
$$= 4 \otimes 4$$
$$= (4+4)^2$$
$$= 64$$

3. Any square adjacent to two squares with different numbers gets the third number. It is easy to fill up the squares. Answer $= \boxed{1321}$.

4. At 6.30, the minute hand is on 6 and the hour hand is exactly in between 6 and 7. There is an angle of $30°$ between 6 and 7 (and more generally, between any two consecutive numbers). So the angle between the hour hand and minute hand is $\boxed{15°}$.

5. There are more cars than tricycles and more tricycles than spaceships. Hence there must be at least one spaceship, at least two tricycles and at least three cars. In this case, the number of wheels is $1 \times 6 + 2 \times 3 + 3 \times 4 = 6 + 6 + 12 = 24$. This exactly equals the number of wheels given. So we must have exactly $\boxed{3}$ cars.

6. To avoid consecutive heads or tails, the only possible patterns are $HTHTH$ and $THTHT$. The probability of each is $\frac{1}{32}$. So the probabilty of either event is $\frac{2}{32} = \boxed{\frac{1}{16}}$.

7. Note that $5 - 12 - 13$ and $9 - 12 - 15$ are Pythagorean triplets. ΔBCD is a right triangle with $\overline{CD} = 5$ and hypotenuse, $\overline{BD} = 13$. Hence $\overline{BC} = 12$. ΔABC is a right triangle with $\overline{AB} = 9$, $\overline{BC} = 12$. Hence, the hypotenuse, $\overline{AC} = \boxed{15}$

8. 80 people purchased oranges and 60 people purchased apples. Since there are 100 people, at least $80 + 60 - 100 = 40$ people purchased both apples and oranges. 70 people purchased bananas. Hence at least $40 + 70 - 100 = \boxed{10}$ people purchased apples, bananas and oranges.

9. We makes cases based on who might be telling the truth.

   If Francis were telling the truth, then according to him, Fred at the cookies. But Fred is making a false statement, hence Ted ate the cookies as well. But we know that only one person ate the cookies. So Francis is not telling the truth.

   If Fred were telling the truth, then Ted is lying. This means Francis is telling the truth. But we know that only one person is telling the truth. So Fred is not telling the truth.

   This leaves the case where Ted is telling the truth. Then Fred is lying, which means $\boxed{Ted}$ ate the cookies.

   Bonus: Why does Ted's name start with a T and Francis' and Fred's names start with F?

10. One way to solve this would be to start listing the words alphabetically. But a faster way would be list them in descending order. Since there are 4 letters, 2 of them identical, the

total number of words is $\dfrac{4!}{2!} = 12$. The 12th word is $TMMB$. The 11th word is $TMBM$. The 10th word is $TBMM$. The 9th word is $\boxed{MTMB}$.

11. ΔABC is right angled at A. So $Area[ABC] = \dfrac{\overline{AC} \times \overline{BC}}{2} = \dfrac{6 \times 8}{2} = 24$. On the other hand, with base BC and height AD, $Area[ABC] = \dfrac{\overline{BC} \times \overline{AD}}{2}$. Hence $AD = \dfrac{2 \times Area[ABC]}{\overline{BC}} = \dfrac{2 \times 24}{10} = \boxed{\dfrac{24}{5}}$. (The length of BC is given by the Pythagoras theorem on ΔABC).

12. Since the number is even the unit's digit must be even. And since there are an even number of even digits, there must be exactly two even digits. We make two cases.

    Case 1: The ten's digit and unit's digit are even. Then the hundred's digit is odd, and can be chosen in 5 ways. Then ten's and unit's digit can be chosen each in 5 ways. So the total number of ways is $5 \times 5 \times 5 = 125$.

    Case 2: The hundred's digit and unit's digit are even. Then the ten's digit is odd and can be chosen in 5 ways. The unit's digit is even and can be chosen in 5 ways. The hundred's digit is even, but we cannot have a 0 in the hundred's digit. Hence there are 4 ways to choose the hundred's digit. The answer is $4 \times 5 \times 5 = 100$.

    The answer is $125 + 200 = \boxed{225}$.

13. Since Alice, Charles and Elizabeth stand in this order, we can write something like ???A???C???E???. Bob and Derek must be adjacent to each other, else they are adjacent to Charlie or at the start/end of the line: in both these cases Felicia's condition cannot be fulfilled. Only possibilities are ABDFCE, ADBFCE, ACFBDE, and ACFDBE, yielding $\boxed{4}$ solutions.

14. Because the shadows are in the same ratio as the heights, we can determine that the ratio of the height to the pole to its shadow (in inches) is $\dfrac{(5 * 12 + 10)}{(14 * 12)} = \dfrac{70}{168} = \dfrac{5}{12}$. The length of the shadow of the pole is equal to $20 + 9 = 29$ feet, or 348 inches. Then the height of the pole is $(348)\dfrac{5}{12} = \boxed{145 \text{ inches}}$.

15. Note that $1458 = 2 \cdot 3^6$. When we throw away a third of the balls, we divide the nubmer by 3 and multiply it by 2. Since 3 divides the original number $1458 = 2 \cdot 3^6$, with the first few steps we get the following numbers $2^2 \cdot 3^5, 2^3 \cdot 3^4, 2^3 \cdot 3^3$... and so on. After 6 steps we get the number 2^7. Now three does not divide the number, but 2 divides it. After 7 steps, we get 1. Thus the total number of steps is $6 + 7 = \boxed{13}$.

16. There are 50 coins, out of which one is a quarter. So there are 49 coins whose values sum up to $82 - 25 = 57$ cents. If there is another quarter among the coins, then we would have 48 coins whose values sum up to $57 - 25 = 32$ cents. This is impossible because 48 coins must have a total value of at least 48. Similarly, if there is a dime among the 49 coins, then we would have 48 coins whose values sum up to $57 - 10 = 47$ cents. This is impossible because 48 coins must have a total value of at least 48. So among the 49 coins, there are only pennies and nickels.

    Let there be p pennies and n nickels. Then the total number of coins equals 49, which means $p + n = 49$. On the other hand, the total value of the coins equals 57, which means $p + 5n = 57$.

Eliminating p, we get $p + 5n - (p + n) = 57 - 49$. Hence $4n = 8$, implying $n = 2$. Thus, $p = 47$. The probability of getting a penny out of the 50 coins if $\dfrac{47}{50} = \dfrac{94}{100} = \boxed{0.94}$.

17. The following is a picture of the figure.
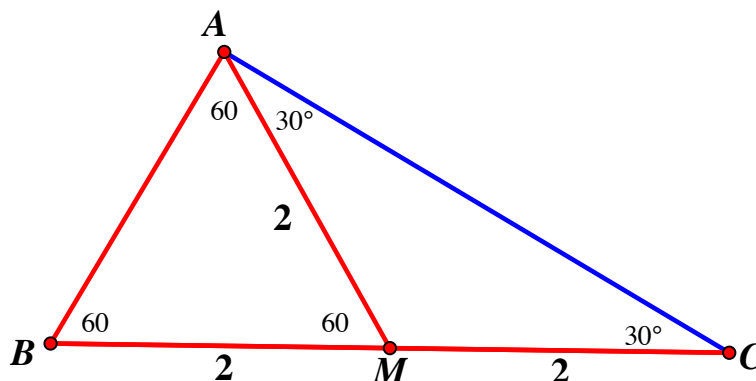
Since $\overline{MA} = \overline{MC}$, ΔAMC is isosceles. Hence $\angle CAM = \angle ACM = 30°$. Then the exterior angle $\angle AMB = \angle ACM + \angle CAM = 30° + 30° = 60°$. Since $\overline{AM} = \overline{MB}$, ΔABM is an isosceles triangle. Since $\angle AMB = 60°$, the triangle is in fact equilateral. Hence $\overline{AB} = 2$.

Now $\angle BAC = \angle BAM + \angle MAC = 60° + 30° = 90°$. By Pythagoras theorem, $\overline{AC}^2 = \overline{BC}^2 - \overline{AB}^2 = 4^2 - 2^2 = 12$. Hence $\overline{AC} = 2\sqrt{3}$. Then $Area[ABC] = \dfrac{\overline{AB} \times \overline{AC}}{2} = \dfrac{2 \times 2\sqrt{3}}{2} = \boxed{2\sqrt{3}}$.

Since $\overline{AB} = \overline{AM} = 5$, ΔABM is an isosceles triangle. So if D is the midpoint of BM, then $AD \perp BM$.

Since M is the midpoint of BC, $\overline{BM} = \overline{MC} = \dfrac{\overline{BC}}{2} = 6$. And since D is the midpoint of BM, $\overline{BD} = \overline{DM} = 3$.

By Pythagoras theorem, $\overline{AD}^2 = \overline{AM}^2 - \overline{DM}^2 = 5^2 - 3^2 = 4^2$. Thus $\overline{AD} = 4$. With base as BC and height as AD, $Area[ABC] = \dfrac{4 \times 12}{2} = 24$.

18. Note $20^n \equiv 1 \bmod 13$ is minimally satisfied for positive integers n when $n = 12$, so the expression $20^n \bmod 13$ may take any value relatively prime to 13. Then, there are $2012 - \lfloor \dfrac{2012}{13} \rfloor = 2012 - 154 = \boxed{1858}$ spirited integers less than 2013.

19. The following is a picture of the figure.
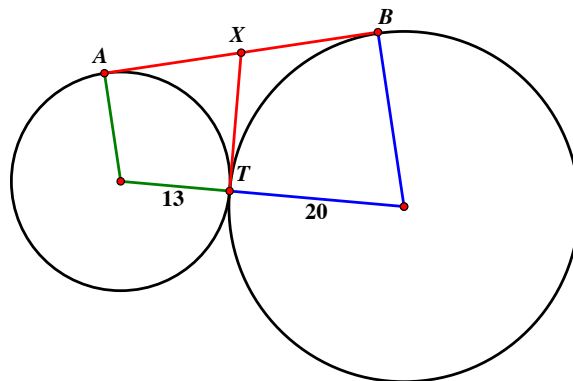
Note $\overline{AX} = \overline{XT} = \overline{XB}$, so $\overline{AX} = \dfrac{1}{2}\overline{AB}$. We calculate $\overline{AB} = \sqrt{(20+13)^2 - (20-13)^2}$, so $\overline{AX} = 2\sqrt{65}$.

20. We see that no admissible set can contain both an odd number and an even number, else the $f(1), f(2), ..., f(n)$ would all be divisible by 2. (This means that no admissible set that satisfies our criteria can contain more than 5 elements.) Similarly, no admissible set can contain numbers that cover all the residues mod 3. Now we examine the cases. When the set has 1 element there are 10 ways to do this. When the set has two elements there are $\binom{5}{2}$ ways to do this for the even elements and $\binom{5}{2}$ ways for the odd elements. When there are 3 elements in the set, there are $\binom{5}{3}$ ways to choose a 3 element set from the even numbers, but we must subtract the 4 sets which contain all three residues mod 3. Thus there are $\binom{5}{3} - 4$ ways for the even numbers and the same number of ways for the odd numbers. There are only two 4-element sets which satisfy the conditions: $\{1, 3, 7, 9\}$ and $\{0, 2, 6, 8\}$. Then the answer is $10 + \left(\binom{5}{2} + \binom{5}{2} \right) + 2 \left(\binom{5}{3} - 4 \right) + 2 = 44$.